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OPTIMAL PUBLIC HARVESTING IN AN ECONOMY WITH MULTIPLE-USE FORESTRY**

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ABSTRACT: This paper studies the design of optimal public harvesting in an economy with multiple-use forestry and private timber supply and where amenity services are public goods. Public and private forest stands are assumed to be either substitutes, independents or complements in the valuation of amenity services. Optimal harvesting is studied both in the situation where the government does care about its revenue, and in the case where social surplus including the government revenue is maximized. In the first case with exogenous timber prices, public harvesting should not be used for ALEP independents and complements, and for ALEP substitutes only if the relative marginal willingness to pay to forego private harvesting is "sufficiently high". Social surplus maximization increases the desirability of public harvesting, ceteris paribus, but public harvesting should not be used in the case of equal marginal willingness to pay to forego harvesting. Allowing for endogenous timber prices increases the desirability of public harvesting when the government revenue is not taken into account. The same happens under social surplus maximization if the public goods aspect of private forests is strong enough.

Keywords: multiple-use, public harvests, marginal willingness to pay
JEL classification: Q23, D26, H21

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TIIVISTELMÄ: Artikkelissa tutkitaan julkisten metsien optimaalista hakkuupolitiikkaa, kun yksityiset ja julkiset metsät tuottavat tuloa ja virkistyspalveluita. Yksityismetsänomistajat reagoivat julkisiin hakkuihin joko suoraan tai kantohintojen muutosten johdosta. Julkisten ja yksityisten metsien oletetaan olevan joko substituutteja, riippumattomia tai komplementteja virkistyskäyttöpalvelujen tuottajina. Julkisten hakkuiden optimaalisuutta tutkitaan, kun vaihtoehtoisesti julkisen sektorin tuloja ei huomioida tai tavoitteena on maksimoida yhteiskunnallista ylijäämää (social surplus). Ensimmäisen tapauksessa olettaen eksogeeniset puun hinnat julkisia hakkuita ei pitäisi suorittaa, jos metsät ovat riippumattomia tai komplementteja virkistyspalveluiden tuottamisessa. Substituuttien tapauksessa julkiset hakkuut ovat suositeltavia vain, jos maksuhalukkuus yksityishakkuista luopumisesta on "riittävän suuri". Yhteiskunnallisen ylijäämän maksimointi lisää julkisten hakkuiden haluttavuutta, mutta jos maksuhalukkuudet hakkuista luopumisesta ovat yhtä suuria, julkisia hakkuita ei tulisi suosia. Puun hintojen endogeenisuuden huomioonottaminen lisää julkisten hakkuiden haluttavuutta, kun julkisen sektorin tuloja ei huomioida. Näin käy myös yhteiskunnallisen ylijäämän maksimoinnin tapauksessa, jos yksityismetsien virkistyspalveluiden julkishyödykeluonne on riittävän voimakas.

Asiasanat: metsien monikäyttö, julkiset hakkuut, maksuhalukkuus
JEL luokitus: Q23,D26,H21.

1. Introduction

Traditional analyses of forest policy have mainly concentrated to study policy issues in the presence of either private timber supply or public forest management. The former analysis has often been carried out by using a two-period intertemporal model, while the public management literature has predominantly used the Faustmann framework.[1] While valuable and important, these traditions neglect the fact that usually both private and public forests coexist and their harvesting may have important interactions, which should be taken into account in designing forest management policy.

In fact, in wood producing countries, the forest land is typically owned by three major owner types, namely, nonindustrial private forest owners, forest firms and the government, but their relative shares considerably differ across countries. In the USA, Finland and Sweden all these ownership types own large amounts of forest land, while in Canada most of the forest land is owned by the government, but some share by firms and others. Moreover, a considerable share of public forests lie in areas, which are generally unfavorable for forest growth. In all these countries, public old growth forests offer greater possibilities for recreational activities and have higher biodiversity than efficiently managed private forests (UN-ECE/FAO 1990 Forest Resource Assessment).

While the goals of forest management across the ownership types may differ, one can, however, argue that the optimal management of all kinds of forests, with a possible exception of the forests owned by firms, takes into account the trade-off between harvesting and recreation. In private forests, the owners value nontimber amenities and thus the harvesting solution reflects the owner's preferences towards amenities (for empirical evidence, see e.g. Binkley 1981, Hyberg and Holthausen 1991, and Kuuluvainen, Karppinen and Ovaskainen 1996). The management of public forests also acknowledges the fact that amenities of forests are a public good, enjoyed by both forest owners and the non-owners of the society. These principles are clearly stated as the goal of national forest services (The UN-ECE/FAO 1990 Forest Resource Assessment) and they are also used as one reason for having an even-flow

[1] See e.g. Montgomery and Adams (1995) for an introduction to and Amacher and Brazee (1997), and Koskela and Ollikainen (1997a) for policy analysis within the framework of the two-period model, and Johansson and Löfgren (1985) for an introduction to, and Boyd and Hyde (1989) for policy analysis within the framework of the Faustmann model. For a simulation analysis of the US Forest Service harvests in the absence of private timber supply response, see Adams et al (1996).

management of public forests.[2]

Forest owners and nonowners (recreationalists in what follows) can enjoy the amenity services produced by both private and public forests. Thus from the viewpoint of amenity services it does not matter whether they go to private or public forests, provided that there are no restrictions or fees for visiting forests.[3] If both public and private forest stands produce amenity services, what is their mutual dependence in the valuation of amenity services by forest owners and recreationalists? In principle, they can be either *substitutes*, *complements* or *independents*. If forests are homogenous in terms of produced amenities, then their relationship is closely tied up with locational aspects: the closer the public forests, the greater the substitutability between private and public forests in terms of recreation and amenities. The case of independents can, however, be developed as well: public forests may be so far away that they do not affect forest owners' and nonowners' valuation. Moreover, in some rare cases also complementarity between private and public forests may be relevant. This occurs, for instance, if a given private forest plot is "sufficiently" small so that the marginal valuation of amenity services provided by a private forest stand increases with the adjacent public forest stand. These intuitive concepts will be given a more formal treatment later on.

The relationship between public and private forests in the valuation of amenities determines how private and public harvesting are 'linked' to each other, i.e., how nonindustrial private forest owners adjust their harvesting to changes in public harvesting.[4] Accounting for the behavioral response of nonindustrial private forest owners to public harvests is important in several respects. First, when public harvesting increases, it decreases the flow of amenities from public forests and the private response may be to decrease (or increase) private harvesting.[5] Second, if public harvesting shifts the aggregate timber supply function, it thereby affects the equilibrium timber prices and harvesting. Third, if the government wants to raise revenue from public harvesting and forest taxes (e.g. yield or site productivity taxes) levied on

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2 For a criticism of even-flow management based on the allowable cut model, see Hyde (1980), and for an analysis of local economic effects, if Forest Service successfully guarantees an even-flow harvest, see Daniels et al (1991).

3 The legislation e.g. in Sweden and Finland guarantees that everyone has right to visit private forests freely according to the so-called every man's right to e.g. walk and gather berries in private forests. In what follows we stick to this assumption for both private and public forests.

4 At this point our analysis comes close to the many-site-multiple-use-problem in which one can analyze how harvests in neighboring forest plots affect the valuation of multiple-use either in many stands (see Bowes and Krutilla 1989) or in a given stand (see Swallow and Wear 1993)

5 The behavioral, incidence and welfare effects of forest taxation under stochastic timber demand but without public harvesting is analyzed in Koskela and Ollikainen (1997b).

nonindustrial private forest owners, how should the forest policy be designed?

The purpose of this paper is to study the optimal public harvest policy in the presence of private and public forests. We use the standard intertemporal multiple-use framework, where both private and public forests produce amenities that are public goods, to study the interaction between private and public harvesting. Private nonindustrial forest owners and recreationalists are assumed to have identical preferences over the amenities from private and public forests. We also explore the implications of endogenous timber prices in designing the optimal forest policy.[6] The rest of the paper is organized as follows. Section 2 presents the theoretical framework for a representative harvest revenue-amenity maximizing forest owner, discusses the marginal valuation of public forest stands, gives relevant comparative statics and develops the timber price effects of public harvesting. Section 3 looks at policy issues. Finally, there are some concluding remarks.

2. PRIVATE TIMBER SUPPLY AND PUBLIC HARVESTS IN A MULTIPLE-USE ECONOMY

2.1 The basic set up

A. Forestry sector and multiple-use

Assume that the representative nonindustrial private forest owner and the government own forest land. Both have an initial stock of timber denoted by Q and Q^g, respectively, where the superscript g refers to the public forests. Denote the current (future) harvesting of private sector and of the government by x (z) and x^g (z^g), respectively. The aggregate joint production of timber and amenities expressed through the stand levels are given in equations [1a]-[1b].

[6] Closest to our analysis is Amacher and Brazee (1996) where the forest tax policy and public harvesting are studied under following assumptions: the government's preferences towards amenity services differ from those of private forest owners, the government has to collect an exogenously given tax revenue via poll and harvest taxes, and the public harvest income is distributed to private forest owners as an exogenous income. Our analysis differs from that of Amacher and Brazee in some respects. We abandon their implicit assumption that public harvesting does not affect private harvesting. Public harvest income is not distributed to private forest owners, but used only as a means of collecting revenue. We also study behavioral and welfare effects of public harvesting with endogenous timber prices.

[1a] $$\hat{k}_1 = k_1 + k_1^g = (Q - x) + (Q^g - x^g),$$

[1b] $$\hat{k}_2 = k_2 + k_2^g = \left[(Q - x) + f(Q - x) - z\right] + \left[(Q^g - x^g) + h(Q^g - x^g) - z^g\right]$$

By harvesting a part x (x^g) of the initial forest stand Q (Q^g) the owners choose also the current aggregate forest stand $\hat{k}_1$ according to [1a], which gives current amenity services. This remaining stock will grow according to concave growth functions $f(Q-x)$ with $f'(\cdot)>0$ and $f''(\cdot)<0$ and $h(Q^g - x^g)$ with $h'(\cdot)>0$ and $h''(\cdot)<0$. We proceed with the simplifying assumption that $f' = h'$. By choosing the size of future harvesting z (z^g) the owners also decide the future aggregate forest stand $\hat{k}_2$ which provides future amenity services. Now $(dk_2 + dz)/dx = -(1+f')<0$ and $(dk_2^g + dz^g)/dx^g = -(1+h')<0$. Thus a rise in current harvesting of either private or public sector means that the sum of future harvesting and future forest stand decreases by the amount which depends on the respective growth function of forests.

Citizens derive utility from the multiple-use of public and private forest stands. The preferences of both forest owners and recreationalists towards amenity services are assumed identical. Given the representative forest owner, the number of recreationalists is n - 1 so that there are n citizens. Recreationalists are assumed to have a preference ordering over their present and future consumption (c_1 and c_2) and over the present and future amenity services provided by the forest stands (k_1, k_1^g and k_2, k_2^g). This is represented by a utility function which is strictly concave in its arguments and additively separable both in consumption and across periods so that $U^R = u(c_1) + R^{-1}u(c_2) + v(k_1, k_1^g) + R^{-1}v(k_2, k_2^g)$, where the time preference factor is assumed to be equal to the interest rate factor $R = (1+r)$. Hence, given the number of citizens, the social valuation of amenity services is given in equation [2].

[2] $$V = n\left(v(k_1, k_1^g) + R^{-1}v(k_2, k_2^g)\right)$$

Thus nontimber benefits are pure public goods with no congestion in public forests, i.e., we assume that citizens can benefit from amenity services without depleting their availability to others.[7]

B. Nonindustrial private forest owner

The nonindustrial private forest owner is assumed to have similar preferences so that

[3] $$U^0 = u(c_1) + R^{-1}u(c_2) + v(k_1, k_1^g) + R^{-1}v(k_2, k_2^g)$$

Thus U^0 describes the discounted utility from consumption and from amenity services of private and public timber stock in both periods.[8] It is assumed that $v_{k_1}, v_{k_1^g}, v_{k_2}, v_{k_2^g}$ are positive for U^R and U^0.[9]

The government levies site productivity tax T on forest owners. The site productivity tax is a lump-sum tax, which is independent of harvesting. In the spirit of traditional public finance we assume that the site productivity tax T is constant over time. During the first period, the forest owner allocates the net revenue from harvesting ($p_1x - T$) plus current non-harvesting income y between consumption (c_1) and saving (s) so that $c_1 = p_1x + y - T - s$. The future consumption is defined by the sum of the future net revenue from harvesting and capital income plus savings minus the site productivity tax so that $c_2 = p_2z - T + Rs$. Combining these flow-of-funds equations yields the intertemporal budget constraint for the forest owner

[4] $$c_2 = p_2z - T + R(p_1x + y - T - c_1) \Leftrightarrow c_1 + R^{-1}c_2 = p_1x + y + R^{-1}p_2z + (1 + R^{-1})T$$

In this framework, public harvesting affects private harvesting behavior only via amenity services of forest stands. In order to analyze the effects of amenity valuation in a sharp focus

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[7] Following the original analysis of Bowes and Krutilla (1985) and (1989), Swallow and Wear (1993) assume, in a modified Hartman model of multiple-use (see Hartman 1976) that amenity value of a given forest stand a, depends on its own age t and the age of neighboring forest stands τ, $a(t, \tau)$. Formulation (2) reflects this feature by assuming that the production of amenity services depends on both private and public forests.

[8] See Ovaskainen (1992) for an original formulation of multiple-use forestry in the two-period model under certainty and Koskela and Ollikainen (1997a) for an analysis of optimal forest taxation under multiple-use forestry and timber price uncertainty.

[9] In what follows the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. E.g. $u'(c_1) = \partial u(c_1) / \partial c_1$, $A_x(x, y) = \partial A(x, y) / \partial x$ etc.

we assume that the forest owner's utility is linear in consumption but nonlinear in amenities.

[5] $$U = p_1 x + y + R^{-1} p_2 z - T(1+R^{-1}) + v(k_1, k_1^g) + R^{-1} v(k_2, k_2^g).$$

In most cases the results presented in this paper are qualitatively similar to those obtained from [3] but the quasi-linear objective function has the advantage of simplifying the analysis and exposition (see Appendix 1 for some basic results from the more general objective function [3]). In the case of recreationalists, we assume correspondingly that $U^R = y + v(k_1, k_1^g) + R^{-1} v(k_2, k_2^g)$.

2. 2 Private Timber Supply and Public Harvests under Multiple-Use Forests

A. Optimal Private Harvesting

The decision problem of the representative nonindustrial private forest owner can now be posed as maximizing the utility U in equation [5] with respect to x and z subject to the intertemporal budget constraint [4], and production of timber equations [1a] and [1b]. The first-order conditions for the utility maximization at the interior solution in the *harvest revenue-amenity model* are

[6a] $$U_x = p_1 - \left[v_{k_1} + R^{-1}(1+f')v_{k_2}\right] = 0$$

[6b] $$U_z = R^{-1} p_2 - R^{-1} v_{k_2} = 0,$$ [10]

where harvesting is made so as to equalize the marginal return (prices) and the marginal costs (the marginal valuation of amenity services). The system of equations [6a] - [6b] implicitly determines current and future timber supply as a function of exogenous parameters, $x = x(T, x^g, z^g, p_1, p_2)$ and $z = z(T, x^g, z^g, p_1, p_2)$.[11]

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[10] In deriving [6a] and [6b], we have applied the chain rule of differentiation and the fact that $\frac{dk_1}{dx} = -1$, $\frac{dk_2}{dx} = -(1+f')$ and $\frac{dk_2}{dz} = -1$. The second-order conditions for the maximum hold and are given in Appendix 2.

[11] Strang (1983) and Swallow and Wear (1993) have pointed out potential convexity problems with such nontimber benefit functions which depend on time in the Faustmann model but in our model it is plausible to assume that amenity services are a smooth function of the total amount of unharvested

Substituting [6b] into [6a] yields the generalized harvesting rule at the interior solution.

[7] $$Rp_1 - p_2(1+f') = Rv_{k_1}$$

According to [7], the valuation of amenity services will have the effect of decreasing harvesting today relative to the case, where only harvest revenue matters.

B. Public Forests and the Marginal Valuation of Amenities

The novel feature of the model is the dependence of private forest owner's utility on the level of public harvests, i.e., the existence of k_1^g as an argument of the valuation function v. Public forest stands cause a positive externality to the private forest owner, but as the first-order condition [7] reveals, the role of the public forest stock for behavioral response depends on the precise nature of the valuation function v.

We use the so-called Auspitz-Lieben-Edgeworth-Pareto (ALEP) definition for complementarity/substitutability of private and public forest stands to analyze the dependence of private and public forests in the valuation of amenities.

Definition: *Private and public forest stands are said to be substitutes, complements or independents in the ALEP sense, when an increase in public harvesting increases, decreases or leaves unchanged, respectively, the marginal utility of a given private forest stand.*[12]

As we mentioned earlier, the relationship between public and private forests in the amenity valuation is closely related to their locations. The ALEP substitutability between amenity services is obtained when private and public forest stands "similar enough and "close enough". If public and private forests are locationally so far away that they become independents. While one would be tempted to regard substitutability or independence as plausible outcomes, there is, however, at least one case where complementarity becomes important. This is when a given forest plot is "sufficiently" small so that recreation requires

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forest.

[12] See Samuelson (1974) and further discussions in Chipman (1977) and in Kannai (1980).

the use of the adjacent public forest stand. Then the owner has to 'enjoy' both stands at the same time. In Appendix 2 we present parametric specifications of amenity valuation which produce each of the possibilities.

2.3 Private response to public harvests and other comparative statics

This section explores how the representative nonindustrial private forest owner reacts to changes in the site productivity tax, public harvesting and timber prices. The site productivity tax has no effects on private harvesting ($x_T = z_T = 0$), and it is straightforward to show that $x_{p_1}, z_{p_2} > 0$ while $x_{p_2}, z_{p_1} < 0$ (see Appendix 3). For changes in current public harvesting one obtains the following private reactions to

[11a] $$x_{x^g} = -J^{-1}\left\{R^{-1} v_{k_1 k_1^g} v_{k_2 k_2}\right\} \le (>) 0 \text{, as } v_{k_1 k_1^g} \le (>) 0$$

[11b] $$z_{x^g} = -J^{-1}\left\{R^{-2} v_{k_2 k_2^g} (1+h') f'' v_{k_2}\right\} \le (>) 0 \text{, as } v_{k_2 k_2^g} \le (>) 0 \text{,}$$

where $J > 0$ (see Appendix 3 for details). The private response to an increase in the current public harvesting depends on the forest owner's marginal valuation of amenity services. If public and private forests are ALEP independents, a change in public harvesting does not affect the marginal valuation of amenities so that there is no change in current and future private harvesting. For ALEP substitutes a rise in public harvesting will decrease private harvesting; a lower level of public forest stand increases the marginal valuation of private forests which induces the forest owner to conserve private forests for the enjoyment of amenity services. Under ALEP complements the reverse happens and the forest owner increases his harvesting.

For future public harvesting we have

[11c] $$x_{z^g} = 0$$

[11d] $$z_{z^g} = -J^{-1}\left\{R^{-1} v_{k_2 k_2^g} [v_{k_1 k_1} + R^{-1} f'' v_{k_2}]\right\} \le (>) 0 \text{, as } v_{k_2 k_2^g} \le (>) 0 \text{.}$$

The independence of current private harvesting from future public harvesting results in turn from the harvesting rule [7] which is independent of future public harvesting. A rise in future public harvesting increases, decreases or leaves future private harvesting unchanged, when forests are ALEP substitutes, complements or independents, respectively.

Thus we have

Result 1: *Private timber supply response to public harvesting depends on the relationship of private and public forests in the amenity valuation, being negative, zero or positive when private and public forests are ALEP substitutes, independents or complements, respectively, with the exception that current private harvesting is independent of future public harvesting.*

2.4 Timber Price Effects of Public Harvesting

What happens to timber prices when public harvesting changes? Aggregate timber supply comes from two sources: private and public harvesting. Let us assume that the firms in the forest industry produce a final product (e.g. pulp) by using roundwood. Production functions are identical for both periods showing decreasing marginal productivity in terms of roundwood input, i.e., $Q_1 = F(x)$ for current and $Q_2 = F(z)$ for future production, respectively with $F' > 0$ and $F'' < 0$. Making the small open economy assumption that the price of the final product is exogenous and normalizing it to one, the decision problem of the firm is to choose x and z so as to maximize the present value of its profits: Max $\pi = F(x) - p_1 x + R^{-1}(F(z) - p_2 z)$. This yields the current and future demand for timber $x^d = x^d(p_1)$ and $z^d = z^d(p_2)$, which depend negatively on the same period's timber price.

The competitive equilibrium in the roundwood market is determined by the equality between timber demand and aggregate timber supply, i.e., $x^d(p_1) = x^s(p_1, p_2, x^g, z^g) + x^g$ and $z^d(p_2) = z^s(p_1, p_2, x^g, z^g) + z^g$, where the superscript s denotes private supply (see Appendix 3).

The price effects of public harvesting can be decomposed into direct and indirect effects. The direct effect is negative, because a rise in public harvesting directly increases aggregate timber supply. The indirect effect depends on how private timber supply reacts to a rise in public harvesting. For ALEP independents there is no reaction, and for ALEP complements private supply increases thus reinforcing the direct negative price effect. Finally, for ALEP substitutes private harvesting decreases thus offsetting, but only partially, the direct price effect of public harvesting (for details, see Appendix 4).

Hence we have

Result 2: *When private forest owners value amenity services provided by private and public forests, current and future public harvesting affect timber prices negatively regardless of whether private and public forests are ALEP independents, complements or substitutes.*[13]

3. FOREST TAXATION AND OPTIMAL PUBLIC HARVESTING

We now turn to consider forest policy issues in a multiple-use economy with private timber supply. Because amenity services of public forests are public goods for all citizens and amenity services of private forests are public goods for recreationalists, there is too much harvesting at the private optimum. Is there any role for public harvesting under these circumstances? If the government decides to harvest, a negative externality is created for both private forest owners and recreationalists via a fall in amenity services. Private forest owners may adjust their harvesting as a response, which in turn may cause another external (positive or negative) effect on recreationalists. Would it then be better to entirely abstain from public harvesting?

Assume that the government chooses a forest policy -- here public harvesting and the site productivity tax -- so as to maximize the social welfare function. The social welfare function consists of the indirect utility function of forest owners and recreationalists (the first and third

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[13] Moreover, one can classify the price effects of public harvesting on both current and future timber price as $\frac{dp_i}{dx^g}|C < \frac{dp_i}{dx^g}|I < \frac{dp_i}{dx^g}|S$, where C, I and S refer to ALEP complements, independents and substitutes, respectively and $i = 1,2$. Not surprisingly, the price effects are qualitatively stronger for complements than independents and stronger for independents than for substitutes.

RHS terms in [12]) and of the indirect profit function of the firms in forest industry (the middle term in [12]) so that

$$[12] \qquad W = U^*(x^g, z^g, p_1, p_2, T, y) + \pi^*(p_1, p_2) + (n-1)\left[y + \left(v(k_1, k_1^g) + R^{-1}v(k_2, k_2^g)\right)\right],$$

where $\pi^*(p_1, p_2)$ is the indirect profit function of firms.

The present value of government revenue from site productivity tax and harvesting is given by

$$[13] \qquad G = (1 + R^{-1})T + p_1 x^g + R^{-1} p_2 z^g .$$

The government is assumed to maximize [12] subject to [13]. Given however that the site productivity tax is neutral (i.e., $x_T = z_T = 0$), this constrained maximization problem reduces that of maximizing the social surplus over the agents target functions and government revenue.[14]

3.1 Maximizing the Social Surplus: Exogenous Timber Prices

Let us consider a case where timber prices are exogenously given though harvesting will change, i.e., the price elasticity of the demand for timber is infinite. The government maximizes the social surplus which is equivalent to maximize

$$[14] \qquad SW = ny + \left[v^*(k_1, k_1^g) + R^{-1}v^*(k_2, k_2^g)\right] + (n-1)\left[v(k_1, k_1^g) + R^{-1}v(k_2, k_2^g)\right] + \left[F(x) + R^{-1}F(z)\right] + \left[p_1 x^g + R^{-1} p_2 z^g\right],$$

where the first term is the non-forest income of citizens, the second one the indirect utility of amenity services for forest owner, the third one is the utility of amenity services for recreationalists, the fourth one is the present value of production and the last term is the

14 To see this write the Lagrangian as $L = W - \lambda(\overline{G} - G)$ for the constraint maximization problem. Choosing the optimal site productivity tax (T) yields $L_T = U_T^* + \lambda G_T = 0$. This is equivalent to $L_T = -(1 + R^{-1}) + \lambda(1 + R^{-1}) = 0$ so that $\lambda = 1$. Thus, in the case of non-distortionary taxation, maximizing [12] subject to [13] is equivalent to the maximization of the social surplus, *SW*=*W*+*G*, because the marginal value of money in the private and the government pocket is the same when $\lambda = 1$.

present value of public harvesting.[15]

Next we define *the marginal willingness to pay to forego public harvesting* for both periods (denoted by w_{x^g} and w_{z^g}) as a change in income which keeps the agents at the previous utility level when public harvesting takes place, i.e., $w_{x^g} = \frac{\partial y}{\partial x^g}$ and $w_{z^g} = \frac{\partial y}{\partial z^g}$.[16] To solve for the measure for the marginal willingness to pay to forego public harvesting the forest owner's utility function is differentiated with respect to x^g , z^g and y while keeping the utility level as constant so that $U^*_{x^g} + \frac{\partial y}{\partial x^g}U^*_y = 0$ and $U^*_{z^g} + \frac{\partial y}{\partial z^g}U^*_y = 0$. Hence we have $w_{x^g} = \frac{\partial y}{\partial x^g} = -\frac{U^*_{x^g}}{U^*_y} = v_{k_1^g} + R^{-1}(1+h')v_{k_2^g}$ and $w_{z^g} = \frac{\partial y}{\partial z^g} = -\frac{U^*_{z^g}}{U^*_y} = R^{-1}v_{k_2^g}$. The corresponding measures for the marginal willingness to pay to forego private harvesting are the market prices so that from [6a]- [6b] we have $w_x = v_{k_1} + R^{-1}(1+f')v_{k_2} = p_1$ and $w_z = R^{-1}v_{k_2} = R^{-1}p_2$.

Choosing current and public harvesting so as to maximize [14] and accounting for that forest owners and firms have maximized with respect to x and z produces the following optimum conditions

[15a] $$SW_{x^g} = -nw_{x^g} - (n-1)\left[p_1 x_{x^g} + R^{-1}p_2 z_{x^g}\right] + p_1 = 0$$

[15b] $$SW_{x^g} = -nw_{z^g} - (n-1)R^{-1}p_2 z_{z^g} + R^{-1}p_2 = 0,$$

where the first two RHS terms describe the marginal costs of public harvesting measured in terms of marginal willingness to pay to forego public harvesting (terms w_{x^g} and w_{x^g}) and to forego private harvesting (terms $-(n-1)\left[p_1 x_{x^g} + R^{-1}p_2 z_{x^g}\right]$ and $-(n-1)R^{-1}p_2 z_{z^g}$), whereas the last terms (p_1 and $R^{-1}p_2$) describe the marginal benefits of public harvesting measured in terms of market prices. Hence, public harvesting should be set so as to equalize the marginal

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[15] Notice that the harvest revenue received by forest owners and timber input costs paid by forest firms cancel out, as well as taxes paid by forest owners and received by the government, when W and G are added together.

[16] See e.g. King (1986). The marginal willingness to pay to forego public harvesting can be measured empirically by the standard techniques of contingent valuation or travel cost methods (see e.g. Bishop et al (1995) and Bockstael (1995)).

costs and benefits.

Consider first the case when the government does not care about its revenue so that the last terms are zero. Then we have from [15a]- [15b]

[16a] $$SW_{x^g|G=0} = 0 \quad \Leftrightarrow \quad -nw_{x^g} - (n-1)\left[p_1 x_{x^g} + R^{-1} p_2 z_{x^g}\right] = 0$$

[16b] $$SW_{z^g|G=0} = 0 \quad \Leftrightarrow \quad -nw_{z^g} - (n-1)R^{-1} p_2 z_{z^g} = 0$$

For ALEP independents and complements $x_{x^g} \geq 0$, $x_{z^g} \geq 0$ so that expressions [16a] and [16b] are negative, i.e., public harvesting should not be used at all. For ALEP substitutes $-1 < x_{x^g}, x_{z^g}, z_{z^g} < 0$ and $z_{x^g} = 0$ so that there are offsetting factors. The interior solution requires that

[16a'] $$SW_{x^g|G=0} = 0 \quad \Leftrightarrow \quad -\left[\frac{p_1}{w_{x^g}} x_{x^g} + \frac{R^{-1} p_2}{w_{x^g}} z_{x^g}\right] = \frac{n}{n-1}$$

[16b'] $$SW_{z^g|G=0} = 0 \quad \Leftrightarrow \quad -\frac{R^{-1} p_2}{w_{z^g}} z_{z^g} = \frac{n}{n-1},$$

where $\frac{n}{n-1} > 1$, $R^{-1} p_2 / w_{x^g} < 1$ and $\left[\frac{p_1}{w_{x^g}} x_{x^g} + \frac{R^{-1} p_2}{w_{x^g}} z_{x^g}\right] > -1$. Now for $p_1 = w_{x^g}$ and $R^{-1} p_2 = w_{z^g}$, we still have a corner solution of zero public harvesting. Only when $p_1 - w_{x^g}$ and $p_2 - w_{z^g}$ are "large enough" an interior solution is obtained.

Hence we have

Proposition 1: *When timber prices are exogenous and government revenue is not taken into account, public harvesting should not be used if private and public forests are ALEP independents or complements in the valuation of amenity services. For ALEP substitutes public harvesting should be used only if the marginal willingness to pay to* *forego private harvesting is "sufficiently" higher than that of public harvesting.*

In the presence of externalities there is too much harvesting at the beginning. Making public harvesting only makes things worse from the viewpoint of amenity valuation of private and public forest stands. Therefore, if the government does not face a budget revenue constraint public harvesting should not be used for ALEP complements and independents. "Sufficiently" high relative valuation of private forests in the case of ALEP substitutes -- measured in terms of marginal willingness to pay to forego private harvesting relative to public harvesting -- means that the decrease in private harvesting more than compensates at the margin the loss of amenities due to public harvesting, thus giving a role for public harvesting as an instrument to correct the inefficiency due to too much private harvesting.

To consider now the social surplus maximization where the government tax revenue is taken into account, write the first-order conditions from [15a] and [15b] as

[17a] $$SW_{x^g} = 0 \quad \Leftrightarrow \quad -\left[\frac{p_1}{w_{x^g}} x_{x^g} + \frac{R^{-1} p_2}{w_{x^g}} z_{x^g}\right] = \frac{1}{n-1}\left[n - \frac{p_1}{w_{x^g}}\right]$$

[17b] $$SW_{z^g} = 0 \quad \Leftrightarrow \quad -\frac{R^{-1} p_2}{w_{z^g}} z_{z^g} = \frac{1}{n-1}\left[n - \frac{R^{-1} p_2}{w_{z^g}}\right]$$

so that accounting for budget revenue requirement increases the desirability of public harvesting. For ALEP independents the LHS terms are zero so that at the interior solution we have $SW_{x^g} = 0 \Leftrightarrow n - (p_1 / w_{x^g}) = 0$ and $SW_{z^g} = 0 \Leftrightarrow n - (R^{-1} p_2 / w_{z^g}) = 0$. If the marginal willingness to pay to forego public and private harvesting is equal, then we have a corner solution in the presence of externalities ($n - 1 > 0$). The same is true for ALEP complements, because now we have some additional negative terms in [17a] and [17b]. As for ALEP substitutes and equal marginal willingness to pay we have $SW_{x^g} = 0$ $\Leftrightarrow -\left[x_{x^g} + \frac{R^{-1} p_2}{w_{x^g}} z_{x^g}\right] = 1$ and $SW_{z^g} = 0 \Leftrightarrow -\frac{R^{-1} p_2}{w_{z^g}} z_{z^g} = 1$ for interior solutions.

Thus we have

Proposition 2: *Under social surplus maximization and exogenous timber prices, public harvesting should not be used regardless of whether private and public forests are ALEP substitutes, independents or complements if the marginal willingness to pay does not differ between public and private forests.*

What could then be the justifications for positive public harvesting in this case? For ALEP independents one must have $n = p_1/w_{x^g} = R^{-1}p_2/w_{z^g}$, i.e., the marginal willingness to pay to forego private harvesting must be higher than that of public harvesting. The same holds for ALEP substitutes. This can be seen from [17a] and [17b]; as p_1/w_{x^g} and $R^{-1}p_2/w_{z^g}$ increase the LHS increases, while the RHS decreases. The case of complements is more complex, since rise in p_1/w_{x^g} and in $R^{-1}p_2/w_{z^g}$ decreases both sides. In [17a] LHS decreases by x_{x^g} and RHS decreases by $1/(n-1)$ and in [17b] LHS decreases by z_{z^g} and RHS decreases by $1/(n-1)$. Even though the public goods nature of amenity services would not be very strong, an interior solution, nevertheless, necessitates that $p_1/w_{x^g} > 1$ and $R^{-1}p_2/w_{z^g} > 1$. Thus we have

Proposition 3: *Under social surplus maximization and exogenous timber prices, public harvesting should be used only if the marginal willingness to pay foregone private harvesting is "sufficiently" high relative to that of public harvesting. This relative willingness to pay should be highest for ALEP complements and lowest for ALEP substitutes.*

3.2 Maximizing the Social Surplus: Endogenous Timber Prices

Let us turn to analyze public harvesting when one accounts for the timber price effects of public harvesting which are relevant under realistic case of less than infinite price elasticity of timber demand. Differentiating the social welfare function [12] and allowing for the timber prices reactions yields the following characterization when the government revenue is not taken into account

[18a] $$SW_{x^g} = SW_{x^g|p^{exog}} - (n-1)\left[\Phi\frac{dp_1}{dx^g} + \Psi\frac{dp_2}{dx^g}\right] = 0$$

[18b] $$SW_{z^g} = SW_{z^g|p^{exog}} - (n-1)\left[\Phi\frac{dp_1}{dz^g} + \Psi\frac{dp_2}{dz^g}\right] = 0\,,$$

where $\Phi = [p_1 x_{p_1} + R^{-1} p_2 z_{p_1}] > 0$ and $\Psi = [p_1 x_{p_2} + R^{-1} p_2 z_{p_2}] > 0$.

Clearly the new RHS terms in [18a]-[18b] are both positive. A rise in public harvesting leads to a fall in current and future timber price. This decreases private harvesting and increases the flow of amenity services for recreationalists. Thereby social welfare goes up.

Maximizing the social surplus [14] leads to the following characterization

[19a] $$SW_{x^g} = SW_{x^g|T=T^*,p^{exog}} + \left(x^g - (n-1)\Phi\right)\frac{dp_1}{dx^g} + \left(R^{-1}z^g - (n-1)\Psi\right)\frac{dp_2}{dx^g} = 0$$

[19b] $$SW_{z^g} = SW_{z^g|T=T^*,p^{exog}} + \left(x^g - (n-1)\Phi\right)\frac{dp_1}{dz^g} + \left(R^{-1}z^g - (n-1)\Psi\right)\frac{dp_2}{dz^g} = 0$$

There are two extra terms in [19a]-[19b] which run counter to each other. On the one hand, a fall in timber prices due to a rise in public harvesting decreases private harvesting thus increasing amenity services of forest stands to recreationalists, which tends to increase public harvesting. But on the other hand, however, a fall in timber prices makes public harvesting less effective as a revenue source for the government, which tends to decrease public harvesting.

Thus we can collect our findings with endogenous timber in

Proposition 4: *Allowing for endogenous timber prices increases the desirability of public harvesting when government revenue is not taken into account. The same happens under social surplus maximization when the public goods aspect of private forests is strong enough.*

4. CONCLUDING REMARKS

This paper has examined the design of optimal public harvesting and forest taxation policy in an economy with multiple-use and private timber supply, when public and private forest stands are either substitutes, independents or complements in the valuation of amenity services. The comparative statics was developed first. The optimal forest policy was then characterized both in the situation where the government does not account for budget constraint and in the situation where social surplus including the government revenue is maximized.

It has been shown that, if the government does not care about budget revenue and timber prices are exogenous, public harvesting should not be used for ALEP independents and complements. For ALEP substitutes public harvesting is optimal when the relative marginal willingness to pay to forego private harvesting is "sufficiently" high. Accounting for budget revenue increases the desirability of public harvesting, ceteris paribus. Public harvesting should not, however, be used in the case of equal marginal willingness to pay to forego (public and private) harvesting. Under social surplus maximization, optimality of public harvesting necessitates that the relative willingness to pay to forego private harvesting should be higher; highest for ALEP complements and lowest for ALEP substitutes.

Allowing for endogeneity of timber prices increases the attractiveness of public harvesting when the government revenue is not taken into account, because introducing public harvesting decreases timber prices, and private harvesting thus increasing the welfare of recreationalists. Under social surplus maximization, allowing for endogenous prices increases the desirability of public harvesting if the public goods aspect of private harvesting is strong enough.

It is interesting to contrast our results to the actual management practice of national Forest Services. Most commonly Forest Services follow the allowable cut model, which leads to an even-flow harvesting. In a multiple-use economy with private timber supply, optimal public harvesting depends on the private timber supply response and the relative willingness to pay to forego public and private harvests which do not feature in the allowable cut model. An interesting topic for further research would be to investigate under what circumstances an even-flow management of public forests would be optimal.

Both the characterization of optimal public harvesting and response of private harvesting to changes in public harvests depends are dependent on the relationship between private and public forests in the valuation of amenity services, i.e., whether public and private forests are substitutes, independents or complements in the private valuation of amenity services. Our analysis was conducted by using a representative forest owner. Hence, given that the relative valuation of private and public forests matters for optimal forest policy, it raises an empirical question: which of the cases, ALEP substitutability, independence or complementarity, is important at the aggregate level? Therefore, it would be important study empirically how private and public forests are related in the private valuation of amenities. Estimating augmented private timber supply function in which public harvests are an additional explanatory variable is a way to approach this empirically unexplored question.

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Appendix 1: The general nonlinear consumption-amenity model (Eq. [3])

The first-order conditions

[1a] $$U_{c_1} = u'(c_1) - u'(c_2) = 0$$

[1b] $$U_x = p_1 u'(c_2) - v_{k_1} - R^{-1}(1+f')v_{k_2} = 0$$

[1c] $$U_z = R^{-1}[p_2 u'(c_2) - v_{k_2}] = 0$$

give the harvesting rule

[2] $$Rp_1 - p_2(1+f') = \frac{Rv_{k_1}}{u'(c_2)}$$

Second-order conditions are

[2a] $$U_{c_1c_1}, U_{xx}, U_{xx} < 0$$

[2b] $$\Delta^1 = \begin{vmatrix} U_{c_1c_1} & U_{c_1x} \\ U_{c_1x} & U_{xx} \end{vmatrix} > 0$$

[2c] $$\Delta = \begin{vmatrix} U_{c_1c_1} & U_{c_1x} & U_{c_1z} \\ U_{c_1x} & U_{xx} & U_{xz} \\ U_{c_1z} & U_{xz} & U_{zz} \end{vmatrix} < 0,$$

Comparative statics

A. Site productivity tax

[3a] $$x_T = \Delta^{-1}\left\{R^{-2}(1+R)u''(c_1)u''(c_2)[Rp_1 - (1+f')p_2]v_{k_2k_2}\right\} > 0$$

[3b] $$z_T = -(1+f')x_T + \phi = ?,$$

where $\phi = \Delta^{-1}\left\{R^{-1}p_2(1+R)u''(c_1)u''(c_2)\left[v_{k_1k_1} + R^{-1}f''v_{k_2}\right]\right\} > 0$.

B. Public harvesting

[4a] $$x_{x^g} = \Delta^{-1}\left\{v_{k_2k_2^g}\left[R^{-1}p_2u''(c_2)^2\Omega - U_{c_1c_1}R^{-1}v_{k_2k_2}\right]\right\} \le (>) 0 \text{ as } v_{k_2k_2^g} \le (>) 0$$

[4b] $$z_{x^g} = -\Delta^{-1}\left\{v_{k_2k_2^g}\left[p_1u''(c_2)^2\Omega + U_{c_1c_1}(f'-h') + f''R^{-1}v_{k_2}\right]\right\} \le (>) 0, \text{ as } v_{k_2k_2^g} \le (>) 0$$

[4c] $$x_{z^g} = \Delta^{-1}\left\{R^{-1}v_{k_2k_2^g}\left[[Rp_1 - p_2(1+f')]u''(c_2)^2\right]\right\} \le (>) 0 \text{ as } v_{k_2k_2^g} \le (>) 0$$

[4d] $$z_{z^g} = -x_{z^g} + \Delta^{-1}\left\{R^{-1}v_{k_2k_2^g}\left[U_{c_1c_1}(v_{k_1k_1} + f''R^{-1}v_{k_2})\right]\right\} \le (>) 0, \text{ as } v_{k_2k_2^g} \le (>) 0$$

where it has been assumed that $\Omega = [Rp_1 - p_2(1+f')](1+h') - p_2 < 0$.

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Appendix 2: Amenity valuation functions for ALEP substitutes, independents and complements

The ALEP substitutability between amenity services can be described by a valuation function

$$[1] \qquad v(k_i, k_i^g) = \begin{cases} \dfrac{(k_i + k_i^g)^{1-\gamma}}{1-\gamma} & as\ \gamma \neq 1 \\ \ln(k_i + k_i^g) & as\ \gamma = 1 \end{cases}$$

where $i = 1,2$. Now one has $v_{k_i k_i} < 0$, $v_{k_i^g k_i^g} < 0$ and $v_{k_i k_i^g} < 0$..

In the case of ALEP independents the valuation function which is additively separable

$$[2] \qquad v(k_i, k_i^g) = v(k_i) + v(k_i^g) \qquad i = 1,2,$$

for which $v_{k_i k_i^g} = 0$ but $v''(k_i) < 0$ and $v''(k_i^g) < 0$.

For ALEP complements the valuation function of amenity services could be described, e.g., by[17]

$$[3] \qquad v(k_i, k_i^g) = \begin{cases} \dfrac{k_i k_i^g}{k_i + k_i^g} \\ or \\ k_i^a (k_i^g)^{1-a} \end{cases}, \qquad i = 1,2,$$

for which one has $v_{k_i k_i} < 0$, $v_{k_i^g k_i^g} < 0$ and $v_{k_i k_i^g} > 0$.

*   *   *   *   *

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17 The first equation is a kind of 'gravity equation', for its derivation see e.g. Anderson (1979) and Bergstrand (1989).

Appendix 3: The quasi-linear harvest revenue-amenity model (Eq. [7])

The first-order conditions are

[1a] $U_x = p_1 - v_{k_1} - R^{-1}(1+f')v_{k_2} = 0$

[1b] $U_z = R^{-1}p_2 - R^{-1}v_{k_2} = 0$

The second-order conditions are

[2a] $U_{xx} = v_{k_1k_1} - R^{-1}(1+f')^2 v_{k_2k_2} + R^{-1}f''v_{k_2} < 0$, $U_{zz} = R^{-1}v_{k_2k_2} < 0$

[2b] $J = \begin{vmatrix} U_{xx} & U_{xz} \\ U_{zx} & U_{zz} \end{vmatrix} = R^{-1}v_{k_2k_2}[v_{k_1k_1} + R^{-1}f''v_{k_2}] > 0$,

where $U_{xz} = U_{zx} = R^{-1}(1+f')v_{k_2k_2} < 0$

Comparative static effects can be solved from the system of equations [1a-1b] - [2a-2c] by applying Cramer's rule, yielding [13a] - [13d] in the text. For section 2.4 we give

[3a] $x_{p_1} = -J^{-1}\{R^{-1}v_{k_2k_2}\} > 0$

[3b] $x_{p_2} = J^{-1}\{R^{-2}(1+f')v_{k_2k_2}\} = -R^{-1}(1+f')x_{p_1} < 0$

[3c] $z_{p_1} = -(1+f')x_{p_1} < 0$

[3d] $z_{p_2} = -(1+f')x_{p_2} + M > 0$,

where $M = -J^{-1}\{R^{-1}[v_{k_1k_1} + R^{-1}f''v_{k_2}]\} = -\frac{1}{v_{k_2k_2}} > 0$

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Appendix 4: The price effects of public harvesting

In this appendix the effects of public harvesting on equilibrium timber prices are derived.

Market equilibrium is defined by

[1a] $x^d(p_1) = x^s(p_1, p_2, x^g, z^g) + x^g$

[1b] $z^d(p_2) = z^s(p_1, p_2, x^g, z^g) + z^g$

Differentiating the equation system [1a]- [1b] with respect to prices and current and future public harvesting yields

[2] $$\begin{bmatrix} x^d_{p_1} - x^s_{p_1} & -x^s_{p_2} \\ -z^s_{p_1} & z^d_{p_2} - z^s_{p_{21}} \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = -\begin{bmatrix} x^s_{x^g} + 1 & x^s_{z^g} \\ z^s_{x^g} & z^s_{z^g} + 1 \end{bmatrix} \begin{bmatrix} dx^g \\ dz^g \end{bmatrix},$$

where the determinant (H) of the coefficient matrix of $\{p_1, p_2\}$ is positive and the sum of the diagonal terms negative so that the system is stable in terms of price adjustments (see e.g. Beavis and Dobbs 1990).

Solving for the effects of current public harvesting produces

[3a] $$\frac{dp_1}{dx^g} = H^{-1}\left\{(1 + x^s_{x^g})[z^d_{p_2} - z^s_{p_2}] + z^s_{x^g} x^s_{p_2}\right\} < 0$$

[3b] $$\frac{dp_2}{dx^g} = H^{-1}\left\{(1 + x^s_{x^g}) z^s_{p_1} + z^s_{x^g}[x^d_{p_1} - x^s_{p_1}]\right\} < 0,$$

where $H = \left\{[x^d_{p_1} - x^s_{p_1}](z^d_{p_2} - z^s_{p_2}) - x^s_{p_2} z^s_{p_1}\right\} > 0$ and $(1 + x^s_{x^g}) > 0$ due to incomplete crowding-out (in the case of ALEP substitutes). The conventional price effects of public harvesting in the absence of amenity valuation are given by $[z^d_{p_2} - z^s_{p_2}] < 0$ and $(z^s_{p_1} < 0)$.

The effects of future harvesting are given by

[3c] $$\frac{dp_1}{dz^g} = H^{-1}\left\{(1 + z^s_{z^g}) x^s_{p_2}\right\} < 0$$

[3d] $$\frac{dp_2}{dz^g} = H^{-1}\left\{(1 + z^s_{z^g})[x^d_{p_1} - x^s_{p_1}]\right\} < 0,$$

respectively. The conventional price effects of public harvesting in the absence of amenity valuation are given by $x^s_{p_2} < 0$ and $([x^d_{p_1} - x^s_{p_1}] < 0)$.

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